



# A2 GCE MATHEMATICS (MEI)

4754/01A Applications of Advanced Mathematics (C4) Paper A

#### **QUESTION PAPER**

Candidates answer on the Printed Answer Book.

#### **OCR** supplied materials:

- Printed Answer Book 4754/01A
- MEI Examination Formulae and Tables (MF2)

#### Other materials required:

• Scientific or graphical calculator

**Duration:** 1 hour 30 minutes

## **INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- Write your answer to each question in the space provided in the Printed Answer Book. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer all the questions.
- Do not write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

### INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.
- This paper will be followed by **Paper B: Comprehension**.

#### INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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### Section A (36 marks)

1 (i) Express  $\frac{x}{(1+x)(1-2x)}$  in partial fractions. [3]

(ii) Hence use binomial expansions to show that  $\frac{x}{(1+x)(1-2x)} = ax + bx^2 + ...$ , where a and b are constants to be determined.

State the set of values of x for which the expansion is valid. [5]

2 Show that the equation  $\csc x + 5 \cot x = 3 \sin x$  may be rearranged as

$$3\cos^2 x + 5\cos x - 2 = 0.$$

Hence solve the equation for  $0^{\circ} \le x \le 360^{\circ}$ , giving your answers to 1 decimal place. [7]

3 Using appropriate right-angled triangles, show that  $\tan 45^\circ = 1$  and  $\tan 30^\circ = \frac{1}{\sqrt{3}}$ .

Hence show that  $\tan 75^\circ = 2 + \sqrt{3}$ .

- 4 (i) Find a vector equation of the line l joining the points (0,1,3) and (-2,2,5). [2]
  - (ii) Find the point of intersection of the line l with the plane x + 3y + 2z = 4. [3]
  - (iii) Find the acute angle between the line *l* and the normal to the plane. [3]
- 5 The points A, B and C have coordinates A(3,2,-1), B(-1,1,2) and C(10,5,-5), relative to the origin O. Show that  $\overrightarrow{OC}$  can be written in the form  $\lambda \overrightarrow{OA} + \mu \overrightarrow{OB}$ , where  $\lambda$  and  $\mu$  are to be determined.

What can you deduce about the points O, A, B and C from the fact that  $\overrightarrow{OC}$  can be expressed as a combination of  $\overrightarrow{OA}$  and  $\overrightarrow{OB}$ ?

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### Section B (36 marks)

6 The motion of a particle is modelled by the differential equation

$$v\frac{\mathrm{d}v}{\mathrm{d}x} + 4x = 0,$$

where x is its displacement from a fixed point, and v is its velocity.

Initially x = 1 and v = 4.

(i) Solve the differential equation to show that 
$$v^2 = 20 - 4x^2$$
. [4]

Now consider motion for which  $x = \cos 2t + 2\sin 2t$ , where x is the displacement from a fixed point at time t.

- (ii) Verify that, when t = 0, x = 1. Use the fact that  $v = \frac{dx}{dt}$  to verify that when t = 0, v = 4. [4]
- (iii) Express x in the form  $R\cos(2t \alpha)$ , where R and  $\alpha$  are constants to be determined, and obtain the corresponding expression for v. Hence or otherwise verify that, for this motion too,  $v^2 = 20 4x^2$ .
- (iv) Use your answers to part (iii) to find the maximum value of x, and the earliest time at which x reaches this maximum value. [3]
- 7 Fig. 7 shows the curve BC defined by the parametric equations

$$x = 5 \ln u$$
,  $y = u + \frac{1}{u}$ ,  $1 \le u \le 10$ .

The point A lies on the x-axis and AC is parallel to the y-axis. The tangent to the curve at C makes an angle  $\theta$  with AC, as shown.

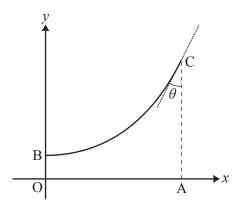


Fig. 7

(i) Find the lengths OA, OB and AC. [5]

(ii) Find 
$$\frac{dy}{dx}$$
 in terms of  $u$ . Hence find the angle  $\theta$ .

(iii) Show that the cartesian equation of the curve is 
$$y = e^{\frac{1}{5}x} + e^{-\frac{1}{5}x}$$
. [2]

An object is formed by rotating the region OACB through 360° about Ox.

(iv) Find the volume of the object. [5]

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## THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE.



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# **A2 GCE MATHEMATICS (MEI)**

**4754/01** Applications of Advanced Mathematics (C4)

**INSTRUCTIONS** 



The examination is in two parts:

Paper A (1 hour 30 minutes)
Paper B (up to 1 hour)

Supervisors are requested to ensure that Paper B is not issued until Paper A has been collected in from the candidates.

Centres may, if they wish, grant a supervised break between the two parts of this examination.

Paper B should not be attached to the corresponding paper A script. For Paper A only the candidates' printed answer books, in the same order as the attendance register, should be sent for marking; the question paper should be retained in the centre or recycled. For Paper B only the question papers, on which the candidates have written their answers, should be sent for marking; the insert should be retained in the centre or recycled. Any additional sheets used must be carefully attached to the correct paper.

For Paper B (Comprehension) only.

A standard English dictionary is allowed for the comprehension.

(Dictionaries to be used in the examination must be thoroughly checked before the examination.) Full regulations are in the JCQ Regulations and Guidance booklet.

This notice must be on the Invigilator's desk at all times during the morning of Thursday 13 June 2013.



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# **A2 GCE MATHEMATICS (MEI)**

4754/01A Applications of Advanced Mathematics (C4) Paper A

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• Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



Candidate forename				Candidate surname			
Centre number				Candidate nu	ımber		

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# Section A (36 marks)

1 (i)	

1 (ii)	

2	

3	
3	

4 (i)	
4 (ii)	

4 (iii)	

_	
5	
1	

# Section B (36 marks)

6 (i)	

6 (ii)	

6 (iii)	

6 (iv)	

7 (i)	

7 (ii)	

7 (iii)	
7 (iv)	
/ (IV)	
	(answer space continued on next page)

7 (iv)	(continued)



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Centre number				Candidate nu	ımber		

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# Section A (36 marks)

1 (i)	

1 (ii)	

2	

3	
3	

4 (i)	
4 (ii)	

4 (iii)	

_	
5	
1	

# Section B (36 marks)

6 (i)	

6 (ii)	

6 (iii)	

6 (iv)	

7 (i)	

7 (ii)	

7 (iii)	
7 (iv)	
	(answer space continued on next page)

7 (iv)	(continued)



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**GCE** 

# **Mathematics (MEI)**

Advanced GCE

Unit 4754A: Applications of Advanced Mathematics: Paper A

# **Mark Scheme for June 2013**

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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# Annotations

Annotation in scoris	Meaning
✓and <b>x</b>	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in	Meaning
mark scheme	
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
WWW	Without wrong working

## Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

#### M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

#### A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

#### В

Mark for a correct result or statement independent of Method marks.

#### $\mathbf{E}$

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep \*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.
  - Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.
- f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or underspecified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.
- g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

	Question		Answer	Marks	Guidance
1	(i)		$\frac{x}{(1+x)(1-2x)} = \frac{A}{1+x} + \frac{B}{1-2x}$		
			$\Rightarrow  x = A(1 - 2x) + B(1 + x)$	M1	expressing in partial fractions of correct form (at any stage) and attempting to use cover up, substitution or equating coefficients Condone a single sign error for M1 only.
			$x = \frac{1}{2} \Rightarrow \frac{1}{2} = B(1 + \frac{1}{2}) \Rightarrow B = \frac{1}{3}$	A1	www cao
			$x = -1 \Rightarrow -1 = 3A \Rightarrow A = -1/3$	A1	www cao
					(accept A/(1+x) +B/(1-2x), $A = -1/3$ , $B = 1/3$ as sufficient for full marks without needing to reassemble fractions with numerical numerators)
				[3]	

Question	Answer	Marks	Guidance
1 (ii)	$\frac{x}{(1+x)(1-2x)} = \frac{-1/3}{1+x} + \frac{1/3}{1-2x}$		
	$= \frac{1}{3} \left[ (1 - 2x)^{-1} - (1 + x)^{-1} \right]$ $= \frac{1}{3} \left[ 1 + (-1)(-2x) + \frac{(-1)(-2)}{2}(-2x)^2 + \dots - (1 + (-1)x + \frac{(-1)(-2)}{2}x^2 + \dots) \right]$	M1	correct binomial coefficients throughout for first three terms of either $(1-2x)^{-1}$ or $(1+x)^{-1}$ oe ie $1,(-1),(-1)(-2)/2$ , not nCr form. Or correct simplified coefficients seen.
	$= \frac{1}{3}[1 + 2x + 4x^2 + \dots - (1 - x + x^2 + \dots)]$	A1	$1+2x+4x^2$
	$\begin{bmatrix} -3 \\ 1 \end{bmatrix}$	A1	$1 - x + x^2$ (or 1/3/-1/3 of each expression, ft their A/B)
			If $k(1-x+x^2)$ (A1) not clearly stated separately, condone absence of inner brackets (ie $1+2x+4x^2-1-x+x^2$ ) <b>only if</b> subsequently it is clear that brackets were assumed, otherwise A1A0.
			[ie $-1-x+x^2$ is A0 unless it is followed by the correct answer]
			Ignore any subsequent incorrect terms
	$= \frac{1}{3}(3x + 3x^2 +) = x + x^2 + \text{ so } a = 1 \text{ and } b = 1$	A1	or from expansion of $x(1-2x)^{-1}(1+x)^{-1}$
	3	AI	www cao
	OR $x(1-x-2x^2) = x(1-(x+2x^2))$ $= x(1+x+2x^2+(-1)(-2)(x+2x^2)^2/2+)$	M1	correct binomial coefficients throughout for (1-(x+2x²)) oe (ie 1,-1), at least as far as necessary terms (1+x) (NB third term of expansion unnecessary and can be ignored)
	$= x (1 + x + 2 x^{2} + x^{2} \dots)$	A2	x(1+x) www
	$= x + x^2 \dots$ so $a = 1$ and $b = 1$	A1	www cao
	Valid for $-\frac{1}{2} < x < \frac{1}{2}$ or $ x  < \frac{1}{2}$	B1	independent of expansion. Must combine as one overall range. condone $\leq$ s (although incorrect) or a combination. Condone also, say $-\frac{1}{2} <  x  < \frac{1}{2}$ but not $x < \frac{1}{2}$ or $-1 < 2x < 1$ or $-\frac{1}{2} > x > \frac{1}{2}$

Question	Answer	Marks	Guidance
2	$\csc x + 5 \cot x = 3 \sin x$		
	$\Rightarrow \frac{1}{\sin x} + \frac{5\cos x}{\sin x} = 3\sin x$	M1	<b>using</b> cosec $x = 1/\sin x$ and cot $x = \cos x / \sin x$
	$\Rightarrow 1 + 5 \cos x = 3 \sin^2 x = 3(1 - \cos^2 x)$	M1	$\cos^2 x + \sin^2 x = 1$ <b>used</b> (both M marks must be part of same solution in order to score both marks)
	$\Rightarrow 3\cos^2 x + 5\cos x - 2 = 0 *$	A1	AG (Accept working backwards, with same stages needed)
	$\Rightarrow (3\cos x - 1)(\cos x + 2) = 0$	M1	use of correct quadratic equation formula (can be an error when substituting into correct formula) or factorising (giving correct coeffs 3 and -2 when multiplied out) or comp square oe
	$\Rightarrow \cos x = 1/3,$	A1	$\cos x = 1/3 \text{ www}$
	$x = 70.5^{\circ},$ $289.5^{\circ}$	A1 A1	for 70.5° or first correct solution, condone over-specification (ie 70.5° or better eg 70.53°,70.5288° etc),
			for 289.5° or second correct solution (condone over-specification) and no others in the range
			Ignore solutions outside the range
			SCA1A0 for incorrect answers that round to 70.5 and 360-their ans, eg 70.52 and 289.48
			SC Award A1A0 for 1.2,5.1 radians (or better)
			Do not award SC marks if there are extra solutions in the range
		[7]	

Question	Answer	Marks	Guidance
3	1 $\tan 45^\circ = 1/1 = 1*$ $\sqrt{3}$ $\tan 30^\circ = 1/\sqrt{3}*$		For both B marks AG so need to be convinced and need triangles but further explanation need not be on their diagram.  Any given lengths must be consistent.
		B1	Need $\sqrt{2}$ or indication that triangle is isosceles oe
		B1	Need all three sides oe
	$\tan 75^{\circ} = \tan (45^{\circ} + 30^{\circ})$	M1	use of <b>correct</b> compound angle formula with 45°,30° soi
	$= \frac{\tan 45 + \tan 30}{1 - \tan 45 \tan 30} = \frac{1 + 1/\sqrt{3}}{1 - 1/\sqrt{3}}$	A1	substitution in terms of $\sqrt{3}$ in any <b>correct</b> form
	$= \frac{1+\sqrt{3}}{-1+\sqrt{3}}$ $= \frac{(1+\sqrt{3})^2}{2}$	M1	eliminating fractions within a fraction (or rationalising, whichever comes first) provided compound angle formula is used as $\tan(A+B) = \tan(A\pm B)/(1\pm \tan A \tan B)$ .
	$(\text{oe eg} \frac{3+\sqrt{3}}{3-\sqrt{3}} = \frac{(3+\sqrt{3})^2}{9-3})$	M1	rationalising denominator (or eliminating fractions whichever comes second)
	$=\frac{(3+2\sqrt{3}+1)}{3-1}=2+\sqrt{3} *$	A1	correct only, AG so need to see working
		[7]	

	Questio	n	Answer	Marks	Guidance
4	(i)		$\mathbf{r} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} + \dots$	B1	need <b>r</b> (or another letter) = or $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ for first B1
			$\dots + \lambda \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$	B1	(-2) $(2)$
					<b>NB</b> answer is not unique eg $\mathbf{r} = \begin{pmatrix} -2 \\ 2 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$
					Accept i/j/k form and condone row vectors.
				[2]	
4	(ii)		x + 3y + 2z = 4		
			$\Rightarrow -2\lambda + 3(1+\lambda) + 2(3+2\lambda) = 4$	M1	substituting their line in plane equation (condone a slip if intention clear)
			$\Rightarrow$ 5 $\lambda$ = -5, $\lambda$ = -1	A1	www cao <b>NB</b> λ is not unique as depends on choice of line in (i)
			so point of intersection is (2, 0, 1)	A1	www cao
				[3]	
4	(iii)		Angle between $-2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ is $\theta$ where	M1	Angle between i+3j+2k and their direction from (i) ft condone a single sign slip if intention clear
			$\cos \theta = \frac{-2 \times 1 + 1 \times 3 + 2 \times 2}{\sqrt{9}\sqrt{14}} = \frac{5}{3\sqrt{14}}$	M1	correct formula (including cosine), with substitution, for these vectors
					condone a single numerical or sign slip if intention is clear
			$\Rightarrow  \theta = 63.5^{\circ}$	A1	www cao (63.5 in degrees (or better) or 1.109 radians or better)
				[3]	

Question	Answer	Marks	Guidance
5	$ \begin{pmatrix} 10 \\ 5 \\ -5 \end{pmatrix} = \lambda \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} $	M1	required form, can be soi from two or more correct equations
	$\Rightarrow 3\lambda - \mu = 10$	M1	forming at least two equations and attempting to solve oe
	$2\lambda + \mu = 5 \Rightarrow 5\lambda = 15,  \lambda = 3$	A1	www
	$\Rightarrow$ 9 - $\mu$ = 10, $\mu$ = -1	A1	www
	$-5 = -\lambda + 2 \mu$ , $-5 = -3 + 2 \times -1$ true	A1	verifying third equation, <b>do not</b> give BOD
			accept a statement such as $\begin{pmatrix} 10\\5\\-5 \end{pmatrix} = 3 \begin{pmatrix} 3\\2\\-1 \end{pmatrix} + -1 \begin{pmatrix} -1\\1\\2 \end{pmatrix}$ as verification
			Must <b>clearly</b> show that the solutions satisfy all the equations.
	coplanar	B1	oe <b>independent</b> of all above marks
		[6]	

	Question	Answer	Marks	Guidance
6	(i)	v dv/dx + 4x = 0		
		$\int v dv = -\int 4x \ dx$	M1	separating variables and intending to integrate
		$1/2 v^2 = -2x^2 + c$	A1	oe condone absence of $c$ . [Not immediate $v^2 = -4x^2$ (+c)]
		When $x = 1$ , $v = 4$ , so $c = 10$	B1	finding c, must be convinced as AG, need to see at least the statement given here oe (condone change of c)
		so $v^2 = 20 - 4x^2 *$	A1	<b>AG</b> following finding <i>c</i> convincingly
				Alternatively, SC $v^2=20-4x^2$ ,
				by differentiation, $2v  dv/dx = -8x$
				v dv/dx + 4x = 0 scores B2
				if, in addition, they check the initial conditions a further B1 is scored (ie 16=20-4). Total possible 3/4.
			[4]	
6	(ii)	$x = \cos 2t + 2\sin 2t$		
		when $t = 0$ , $x = \cos 0 + 2 \sin 0 = 1*$	B1	AG need some justification
		$v = dx/dt = -2\sin 2t + 4\cos 2t$	M1	differentiating, accept $\pm 2,\pm 4$ as coefficients but not $\pm 1,\pm 2$ and not $\pm 1/2,\pm 1$ from integrating
			A1	cao
		$v = 4 \cos 0 - 2\sin 0 = 4*$	A1	www AG
			[4]	

Question	Answer	Marks	Guidance
6 (iii)	$\cos 2t + 2\sin 2t = R\cos(2t - \alpha) = R(\cos 2t \cos \alpha + \sin 2t \sin \alpha)$		SEE APPENDIX 1 for further guidance
	$R = \sqrt{5}$	B1	or 2.24 or better (not $\pm$ unless negative rejected)
	$R\cos\alpha=1, R\sin\alpha=2$	M1	correct pairs soi
	$\tan \alpha = 2$ ,	M1	correct method
	$\alpha = 1.107$	A1	cao radians only, 1.11 or better (or multiples of $\pi$ that round to 1.11)
	$x = \sqrt{5}\cos(2t - 1.107)$		
	$v = -2\sqrt{5}\sin(2t - 1.107)$	A1	differentiating or otherwise, ft their numerical $R$ , $\alpha$ (not degrees) required form SC B1 for $v = \sqrt{20} \cos(2t + 0.464)$ oe
	<b>EITHER</b> $v^2 = 20\sin^2(2t - \alpha)$		
	$20 - 4x^2 = 20 - 20\cos^2(2t - \alpha)$	M1	squaring their $v$ (if of required form with same $\alpha$ as $x$ ), and $x$ , and attempting to show $v^2 = 20 - 4x^2$ ft their $R$ , $\alpha$ (incl. degrees) [ $\alpha$ may not be specified].
	$= 20(1 - \cos^{2}(2t - \alpha)) = 20\sin^{2}(2t - \alpha)$ so $v^{2} = 20 - 4x^{2}$	A1	cao www (condone the use of over-rounded $\alpha$ (radians) or degrees)
	OR multiplying out $v^2 = (-2\sin 2t + 4\cos 2t)^2$ = $4\sin^2 2t - 16\sin 2t\cos 2t + 16\cos^2 2t$ and $4x^2 = 4(\cos^2 2t + 4\sin 2t\cos 2t + 4\sin^2 2t)$ = $4\cos^2 2t + 16\sin 2t\cos 2t + 16\sin^2 2t$ (need middle term) and attempting to show that $v^2 + 4x^2 = 4(\sin^2 2t + \cos^2 2t) + 16(\cos^2 2t + \sin^2 2t)$ = $4+16 = 20$ (or $20-4x^2 = v^2$ ) oe	M1	differentiating to find $v$ (condone coefficient errors), squaring $v$ and $x$ and multiplying out (need attempt at middle terms) and attempting to show $v^2 = 20 - 4x^2$
	so $v^2 = 20 - 4x^2$	A1	cao www
		[7]	

(	Questio	n	Answer	Marks	Guidance
6	(iv)		$x = \sqrt{5}\cos(2t - \alpha)$ or otherwise		
			$x \max = \sqrt{5}$	B1	ft their R
			when $cos(2t - \alpha) = 1$ , 2t - 1.107 = 0, 2t = 1.107	M1	oe (say by differentiation) ft their $\alpha$ in radians or degrees for method only
			t = 0.55	A1	cao (or answers that round to 0.554)
				[3]	
7	(i)		$u = 10, x = 5 \ln 10 = 11.5$	M1	Using $u = 10$ to find OA
			so $OA = 5 \ln 10$	A1	accept 11.5 or better
			when $u = 1$ ,	M1	Using $u = 1$ to find OB or $u = 10$ to find AC
			y = 1 + 1 = 2 so OB = 2	A1	
			When $u = 10$ , $y = 10 + 1/10 = 10.1$		
			So $AC = 10.1$	A1	
					In the case where values are given in coordinates instead of OA=,OB=,AC=, then give A0 on the first occasion this happens but allow subsequent As.  Where coordinates are followed by length eg B(0, 2), length=2
					then allow A1.
				[5]	

	Questio	on Answer	Marks	Guidance		
7	(ii)	$\frac{\mathrm{d}y}{\mathrm{d}y} = \frac{\mathrm{d}y/\mathrm{d}u}{\mathrm{d}u} = \frac{1 - 1/u^2}{\mathrm{d}u^2}$	M1	their dy/du /dx/du		
		$\frac{1}{dx} = \frac{1}{dx/du} = \frac{1}{5/u}$	A1			
		$\left[ = \frac{u^2 - 1}{5u} \right]$		Award A1 if <b>any</b> correct form is seen at any stage including unsimplified (can isw)		
		EITHER				
		When $u = 10$ , $dy/dx = 99/50 = 1.98$	M1	substituting u =10 in their expression		
		$\tan (90 - \theta) = 1.98 \Rightarrow \theta = 90 - 63.2$	M1	or by geometry, say using a triangle and the gradient of the line		
		= 26.8°	A2	26.8°, or 0.468 radians (or better) cao		
				SC M1M0A1A0 for 63.2° (or better) or 1.103 radians(or better)		
		OR				
		When $u = 10$ , $dy/dx = 99/50 = 1.98$	M1			
		$\tan(90 - \theta) = 99/50 \Rightarrow \tan\theta = 50/99$	M1	allow use of their expression for M marks		
		$\theta = 26.8^{\circ}$	A2	26.8°, or 0.468 radians (or better) cao		
			[6]			
7	(iii)	$x = 5 \ln u \Rightarrow x/5 = \ln u, u = e^{x/5}$ $\Rightarrow y = u + 1/u = e^{x/5} + e^{-x/5}$	M1	Need some working		
		$\Rightarrow y = u + 1/u = e^{x/5} + e^{-x/5}$	A1	Need some working as AG		
			[2]			

Question	Answer	Guidance	
7 (iv)	Vol of rev = $\int_0^{5\ln 10} \pi y^2 dx = \int_0^{5\ln 10} \pi (e^{x/5} + e^{-x/5})^2 dx$	M1	need $\pi (e^{x/5} + e^{-x/5})^2$ and $dx$ soi. Condone wrong limits or omission of limits for M1. Allow M1 if $y$ prematurely squared as eg $(e^{2x/5} + e^{-2x/5})$
	$=\int_0^{5\ln 10} \pi (e^{2x/5} + 2 + e^{-2x/5}) dx$	A1	including <b>correct</b> limits at some stage (condone 11.5 for this mark)
	$= \int_0^{5\ln 10} \pi (e^{2x/5} + 2 + e^{-2x/5}) dx$ $= \pi \left[ \left( \frac{5}{2} e^{2x/5} + 2x - \frac{5}{2} e^{-2x/5} \right) \right]_0^{5\ln 10}$	B1	$\left[\frac{5}{2}e^{2x/5} + 2x - \frac{5}{2}e^{-2x/5}\right]$ allow if no $\pi$ and/or no limits or incorrect limits
	$=\pi(250+10\ln 10-0.025-0)$	M1	substituting both limits (their OA and 0) in an expression of correct form ie $ae^{2x/5} + be^{-2x/5} + cx$ , $a,b,c \neq 0$
			and subtracting in correct order (- 0 is sufficient for lower limit) Condone absence of $\pi$ for M1
	= 858	A1	accept $273\pi$ and answers rounding to $273\pi$ or $858$
		[5]	NB The integral can be evaluated using a change of variable to u. This involves changing $dx$ to $(dx/du)x$ $du$ . For completely correct work from this method award full marks. Partially correct solutions must include the change in $dx$ . If in doubt consult your TL.
			Remember to indicate second box has been seen even if it has not been used.

#### **APPENDIX 1**

ADDITIONAL GUIDANCE for 6(iii)

 $R\cos\alpha=1$ ,  $R\sin\alpha=2$ ,  $R=\sqrt{5}$ ,  $\alpha=1.107$ 

## 1) Missing R

Ie  $\cos\alpha=1$ ,  $\sin\alpha=2$ .

We reluctantly condone this provided that it is followed by working that suggests R was implied such as  $\tan\alpha = 2$ ,  $\alpha = 1.107$  M1M1A1. Other methods are possible.

We do not award M1 for  $\cos\alpha=1$ ,  $\sin\alpha=2$  if it is followed by  $\alpha=\text{inv}\cos 1$  as R is not implied.

B1 is still available.

## 2)Incorrect pairs

eg Rsin $\alpha$ =1,Rcos $\alpha$ =2 scores M0 but would obtain the second M1ft if it was followed by tan $\alpha$ =1/2. M0M1A0. B1 is possible.

## 3)Incorrect method

Rsin $\alpha$ =2, Rcos $\alpha$ =1 followed by tan $\alpha$ =1/2 scores M1M0A0. B1 is possible.

## 4)Incorrect pairs and incorrect method

 $R\sin\alpha=1$ ,  $R\cos\alpha=2$ ,  $\tan\alpha=2$  is M0M0A0. B1 is possible. This is easily over-looked and is a double error leading to an apparently correct answer.

# 5) Incorrect signs (all could score B1)

- (a)  $R\cos\alpha = 1$ ,  $R\sin\alpha = -2$ ,  $\tan\alpha = -2$ , M1, M1ft, A0
- (b)  $R\cos\alpha = 1$ ,  $R\sin\alpha = -2$ ,  $\tan\alpha = 2$ , M1 M0ft, A0 sign error
- (c) Rcos $\alpha$ = 1, Rsin $\alpha$ =-2, R= $\sqrt{5}$ , sin $\alpha$ =-2/ $\sqrt{5}$ , M1 M1 A0 sign error
- (d) Rcos  $\alpha$ =1, Rsin $\alpha$ =-2, sin $\alpha$ = 2/ $\sqrt{5}$ , M1M0 sign error A0
- (e) Rcos $\alpha$  = 1, Rsin $\alpha$ =-2, cos $\alpha$ =1/ $\sqrt{5}$ ,  $\alpha$ =1.107 M1,M1, A0 sign error (even though not used)

# 6) Incorrect R

- a)  $R\cos\alpha=1$ ,  $R\sin\alpha=2$ , R=5 (say),  $\cos\alpha=1/5$ , scores B0 M1M1ftA0
- b)  $R\cos\alpha = 1$ ,  $R\sin\alpha = 2$ , R=5 (say),  $\tan\alpha = 2$ ,  $\alpha = 1.107$  scores M1M1B0A1 (allow)

# 7) Missing Working

- a)  $\tan\alpha=2,\alpha=1.107$ ,  $R=\sqrt{5}$ , scores M1M1A1B1 soi
- b)  $\tan\alpha = 1/2$ ,  $R = \sqrt{5}$  scores M1M0B1A0 (either correct pairs or correct method but not both)
- c) Rcos $\alpha$ =1, R= $\sqrt{5}$ , $\alpha$ =1.107 M1M1B1A1 soi

Other options are possible. Examiners should consult their Team Leaders if in doubt.

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**GCE** 

# **Mathematics (MEI)**

Advanced GCE A2 7895-8

Advanced Subsidiary GCE AS 3895-8

# **OCR Report to Centres**

**June 2013** 

# 4754 Applications of Advanced Mathematics (C4)

#### **General Comments**

This paper was of a similar standard to previous years.

The questions were accessible to candidates of all abilities who were able to demonstrate their skills. There were few very low scores and also few very high scores, with full marks obtained by a few candidates. The higher scoring candidates were able to show their skills - particularly in Paper A questions 3, 6(iii) and 7.

The comprehension, Paper B, was well understood and most candidates scored good marks here.

As in previous years, many candidates lost unnecessary marks through poor algebra. Some particularly common such examples being:

• 
$$\frac{1}{3(1+x)} = 3(1+x)^{-1} = 3(1-x+x^2...)$$

• 
$$\left(e^{x/5} + e^{-x/5}\right)^2 = e^{2x/5} + e^{-2x/5}$$

• 
$$\csc x + 5 \cot x = 3 \sin x \Rightarrow \csc^2 x + 25 \cot^2 x = 9 \sin^2 x$$

These, and other algebraic errors, are detailed later in this Report.

Sign errors also continue to be a common cause of an unnecessary loss of marks.

In contrast, however, it was very pleasing to note that, unlike in previous years, few candidates failed to put a constant of integration. Examiners would now like to encourage candidates to change the constant when say multiplying through by 2 rather than renaming their constant as c at every stage.

Candidates should be reminded that when they are asked to 'Show' they need to show all stages of working. This is improving, but it is disappointing when marks are lost in this way.

Centres should be reminded that Papers A and B are marked separately and so supplementary sheets should be attached to the appropriate paper.

#### **Comments on Individual Questions**

#### Paper A

- 1)(i) Whilst almost all candidates knew the general method for expressing the given fraction
- 1)(ii) in partial fractions, there were a surprising number of numerical errors.

Most candidates were able to use the binomial expansion correctly although there were sign errors - often from using (-2x) as (2x).

The most common error-which was very common- was using

$$\frac{1}{3(1+x)} = 3(1+x)^{-1} = 3(1-x+x^2...) = 3-3x+3x^2 \quad \text{and similarly for } \frac{1}{3(1-2x)}.$$

The other frequent error was in the validity. Some candidates omitted this completely but many others failed to combine the validities from the two expansions, or failed to choose the more restrictive option.

2) Many candidates scored full marks when showing that the trigonometric equation could be rearranged as a quadratic and then solving it.

Where there were errors, these were usually in the first part when trying to establish the given result. Errors included failing to use the correct trigonometric identities, failing to use  $\sin^2\theta + \cos^2\theta = 1$  or squaring the original expression term by term. Few candidates would say x+3=7 so  $x^2+9=49$  and yet they happily square cosec x+5cot x=3sin x term by term.

Those who were unable to complete the first part sensibly then proceeded to solve the quadratic equation. Few errors were seen here. Occasionally the final solution was incorrect and few candidates offered additional incorrect solutions.

3) There were some good explanations with appropriate triangles in the first part.

However, too many candidates felt it was enough to only give the information given in the question and this was not sufficient. More was needed than, for example, a right-angled triangle with lengths of 1, 1 and  $45^{\circ}$  to show that tan  $45^{\circ}=1$ . It was necessary to clearly show the triangle was isosceles by giving the other angle or showing that the hypotenuse was  $\sqrt{2}$ , or equivalent. Some made errors when calculating the other lengths in both triangles. Some good candidates failed to score here seemingly being unfamiliar with where these identities came from.

The second part started well for most candidates, who usually used the correct compound angle formula, (although there were a few who thought that tan75°=tan45°+tan30°) and made the first substitution. Thereafter, this question gave the opportunity for candidates to show that they could eliminate fractions within fractions and rationalise the denominator. This was a good discriminator for the higher scoring candidates. A few candidates abandoned their attempt at half way and equated

$$\frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}$$
 at that stage to the given answer 2+ $\sqrt{3}$ .

4) Most candidates scored high marks throughout this question.

In (i) the most common error was to omit r =at the start. Few candidates would write, for instance, y = x+3 without the y but the r =is too often omitted from vector equations.

In (ii) errors were usually numerical and in (iii) they were either numerical errors or the wrong vectors.

5) Most candidates scored the first four marks by forming the equations and solving them.

Marks were usually lost both when candidates failed to show their solutions worked in all three equations or failed to realise that O, A, B and C must all lie on the same plane for the final mark.

6)(i) Most candidates scored all four marks when solving the differential equation. It was pleasing to see so few candidates failing to include the constant of integration. Some candidates, however, tried to work backwards from the answer, or wrote  $v^2 = -4x^2 + c$  without showing from where it came. The answer was given in this case so stages of working were needed.

Whilst, on this occasion, examiners condoned the change of constant candidates should be encouraged to change their constant when appropriate in the future and not use *c* twice to mean different things within the same question.

- 6)(ii) Most candidates obtained the mark for verifying that *x* =1. Many others also scored the following three marks but some had the incorrect coefficients when differentiating and only had the correct coefficient in the second term when working backwards from the answer, 4.
- 6)(iii) This part was rarely answered completely successfully. Most candidates understood that the 'R' method was needed and scored the first three marks. This was the modal mark.

Calculus was needed in this question. The candidates were asked to find the constant  $\alpha$ , and t is a time to be combined in  $(2t-\alpha)$  so answers given in radians were required. The use of degrees here was a very common mistake. Many candidates then differentiated their angles in degrees and obtained no marks for v.

The last two marks were obtained only rarely but they were a good differentiator for the able candidates. Use of unspecified  $\alpha$ , or in degrees or radians was allowed in this last part. Some candidates had difficulty as they had found both x and v separately using the 'R' method and so had different values for their angles. Others realised the problem and were able to use trigonometric identities to change their v (or x) to have the same angle.

- 6)(iv) The majority of candidates scored the first two marks.. It was disappointing that candidates did not realise their mistake in part (ii) when they obtained an answer of time, t=31.7 degrees.
- 7)(i) Some candidates were able to score full marks here with ease. Some candidates gave their answers as coordinates instead of lengths and others found OC instead of AC. There were also, however, some very confused and unclear methods used and many candidates lost marks having failed to use u = 1 and u = 10 or equivalent.
- 7)(ii) Most candidates understood that they needed to find and divide dy/du by dx/du. There was some very poor algebra when attempting to simplify  $\frac{1-1/u^{-2}}{5/u}$ . The derivatives were also frequently wrong-often including ln u. Many candidates stopped at this stage or substituted u =10 in their derivative and then stopped. Some candidates, who were able to score marks in the following stages, failed to realise that they could invert the derivative and quickly find the answer. Some used the gradient, 1.98, to form an equation of a straight line and find its intercept with the x axis. Other candidates, unfortunately, felt they could use 1.98 as a hypotenuse in the triangle with AC-with no success.
- 7)(iii) This was well understood but candidates lost marks by giving insufficient working when establishing a given result.

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7)(iv) There were some excellent solutions here but the majority only scored one mark. This was awarded to those who correctly showed us their intention to find  $V = \int \pi (e^{x/5} + e^{-x/5})^2 dx$ . However, the majority could not expand this bracket. Usually it was thought to equal  $e^{2x/5} + e^{-2x/5}$  but other incorrect options were seen, including powers such as  $x^2/25$ . For those who did expand the bracket correctly, other errors followed-either using the wrong upper limit, failing to substitute the lower limit or, more commonly integrating either 2 as 0, or more particularly  $e^{2x/5}as\frac{2e^{2x/5}}{5}$  and similarly for the other term.

#### Paper B

- 1) Answers were often correct but surprisingly many, and various, incorrect positions were seen. A number of candidates only used the letter *R* or *M* to indicate their points where the addition of a cross or dot would have made their position clearer.
- 2) Most candidates had the right idea but some were inaccurate with 10, 6, 4 being the most common alternative solution.
- 3) The graphs were usually identified correctly although there were also many guesses. The response tended to be either fully correct or all wrong.
- 4) Most candidates substituted  $\alpha = 60^{\circ}$  and found t = 9.2449 or similar and then the majority multiplied it by 2 to compare it with 18. A few worked backwards from t = 9 to reach approximately  $60^{\circ}$  when substituting in the appropriate equation, and were given full credit.
- 5(i) Most candidates found  $\alpha = -17.31$  as required. A few chose to use the number of days in January as 30 and lost one mark. Those who thought February was the first or third month received no credit.
- 5)(ii) Some candidates carelessly lost the negative sign in their angle or the negative sign in the formula and so lost unnecessary marks.

Many correctly obtained t=4.37 but not all converted this correctly to the 24 hour clock. 16:37 was commonly seen.

Candidates were able to follow through for full marks from the Special Case in part(ii)



# Unit level raw mark and UMS grade boundaries June 2013 series AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

GCE Mathematics (MEI)								
		Max Mark	а	b	С	d	е	u
4751/01 (C1) MEI Introduction to Advanced Mathematics	Raw	72	62	56	51	46	41	0
47F0/04 (OO) MFI O	UMS	100	80	70	60	50	40	0
4752/01 (C2) MEI Concepts for Advanced Mathematics	Raw UMS	72 100	54 80	48 70	43 60	38 50	33 40	0 0
4753/01 (C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	58	52	46	40	33	0
4753/02 (C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753/82 (C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4753 (C3) MEI Methods for Advanced Mathematics with Coursework	UMS	100	80	70	60	50	40	0
4754/01 (C4) MEI Applications of Advanced Mathematics	Raw	90	66	59	53	47	41	0
	UMS	100	80	70	60	50	40	0
4755/01 (FP1) MEI Further Concepts for Advanced Mathematics	Raw UMS	72 100	63 80	57 70	51 60	45 50	40 40	0
4756/01 (FP2) MEI Further Methods for Advanced Mathematics	Raw	72	61	54	48	42	36	0
	UMS	100	80	70	60	50	40	0
4757/01 (FP3) MEI Further Applications of Advanced Mathematics	Raw	72	60	52	44	36	28	0
47E0/04 /DE) MELDifferential Equations with Coursewelly Written Denor	UMS	100	80	70	60	50	40	0
4758/01 (DE) MEI Differential Equations with Coursework: Written Paper 4758/02 (DE) MEI Differential Equations with Coursework: Coursework	Raw Raw	72 18	62 15	56 13	51 11	46 9	40 8	0 0
4758/82 (DE) MEI Differential Equations with Coursework: Coursework 4758/82 (DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4758 (DE) MEI Differential Equations with Coursework	UMS	100	80	70	60	50	40	0
4761/01 (M1) MEI Mechanics 1	Raw	72	57	49	41	33	25	0
	UMS	100	80	70	60	50	40	0
4762/01 (M2) MEI Mechanics 2	Raw	72	50	43	36	29	22	0
	UMS	100	80	70	60	50	40	0
4763/01 (M3) MEI Mechanics 3	Raw	72	64	56 70	48	41	34	0
4764/04 (M4) MEL Machanica 4	UMS	100 72	80 56	70 49	60 42	50 35	40 29	0
4764/01 (M4) MEI Mechanics 4	Raw UMS	100	80	49 70	42 60	50	40	0 0
4766/01 (S1) MEI Statistics 1	Raw	72	55	48	41	35	29	0
	UMS	100	80	70	60	50	40	0
4767/01 (S2) MEI Statistics 2	Raw	72	58	52	46	41	36	0
	UMS	100	80	70	60	50	40	0
4768/01 (S3) MEI Statistics 3	Raw	72	61	55	49	44	39	0
	UMS	100	80	70	60	50	40	0
4769/01 (S4) MEI Statistics 4	Raw UMS	72 100	56 80	49 70	42 60	35 50	28 40	0 0
4771/01 (D1) MEI Decision Mathematics 1	Raw	72	58	52	46	40	35	0
477 701 (D1) INCLUDED SIGN Mathematics 1	UMS	100	80	70	60	50	40	0
4772/01 (D2) MEI Decision Mathematics 2	Raw	72	58	52	46	41	36	0
	UMS	100	80	70	60	50	40	0
4773/01 (DC) MEI Decision Mathematics Computation	Raw	72	46	40	34	29	24	0
	UMS	100	80	70	60	50	40	0
4776/01 (NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	56	50	44	38	31	0
4776/02 (NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	/ 7	0
4776/82 (NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark 4776 (NM) MEI Numerical Methods with Coursework	Raw UMS	18 100	14 80	12 70	10 60	8 50	40	0 0
4777/01 (NC) MEI Numerical Computation	Raw	72	55	47	39	32	25	0
177701 (110) MET Hamonodi Compatation	UMS	100	80	70	60	50	40	0
4798/01 (FPT) Further Pure Mathematics with Technology	Raw	72	57	49	41	33	26	0
	UMS	100	80	70	60	50	40	0
GCE Statistics (MEI)		Max Mark	а	b	С	d	е	u
G241/01 (Z1) Statistics 1	Raw	72	55	48	41	35	29	0
22.770. (21) oldiolog 1	UMS	100	80	70	60	50	40	0
G242/01 (Z2) Statistics 2	Raw	72	55	48	41	34	27	0
	UMS	100	80	70	60	50	40	0
G243/01 (Z3) Statistics 3	Raw	72	56	48	41	34	27	0
	UMS	100	80	70	60	50	40	0



# **Thursday 13 June 2013 – Morning**

# **A2 GCE MATHEMATICS (MEI)**

**4754/01B** Applications of Advanced Mathematics (C4) Paper B: Comprehension **QUESTION PAPER** 

Candidates answer on the Question Paper.

#### **OCR** supplied materials:

- Insert (inserted)
- MEI Examination Formulae and Tables (MF2)

#### Other materials required:

- Scientific or graphical calculator
- Rough paper

**Duration:** Up to 1 hour



Candidate forename				Candidate surname			
Centre number				Candidate nu	ımber		

#### **INSTRUCTIONS TO CANDIDATES**

- The Insert will be found in the centre of this document.
- Write your name, centre number and candidate number in the boxes above. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer all the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Write your answer to each question in the space provided. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do not write in the bar codes.
- The Insert contains the text for use with the questions.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

#### **INFORMATION FOR CANDIDATES**

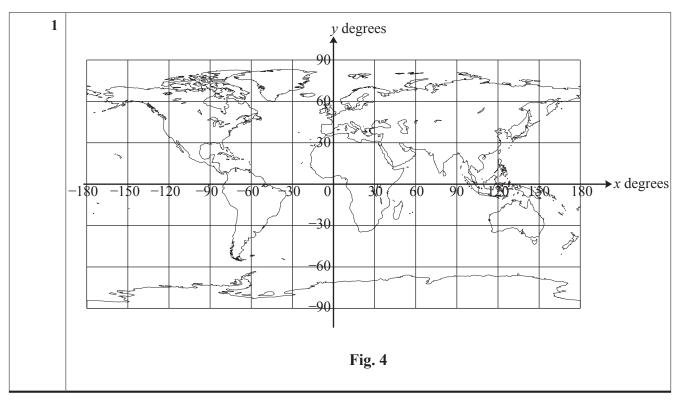
- The number of marks is given in brackets [ ] at the end of each question or part question.
- You may find it helpful to make notes and to do some calculations as you read the passage.
- You are **not** required to hand in these notes with your Question Paper.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 18.
- This document consists of 8 pages. Any blank pages are indicated.



1 The diagram is a copy of Fig. 4.

R is a place with latitude 45° north and longitude 60° west. Show the position of R on the diagram.

M is the sub-solar point. It is on the Greenwich meridian and the declination of the sun is  $+20^{\circ}$ . Show the position of M on the diagram. [2]



Use Fig. 8 to estimate the difference in the length of daylight between places with latitudes of 30° south and 60° south on the day for which the graph applies. [3]

2	

# **3** The graph is a copy of Fig. 6.

The article says that it shows the terminator in the cases where the sun has declination  $10^{\circ}$  north,  $1^{\circ}$  north,  $5^{\circ}$  south and  $15^{\circ}$  south.

Identify which curve (A, B, C or D) relates to which declination.



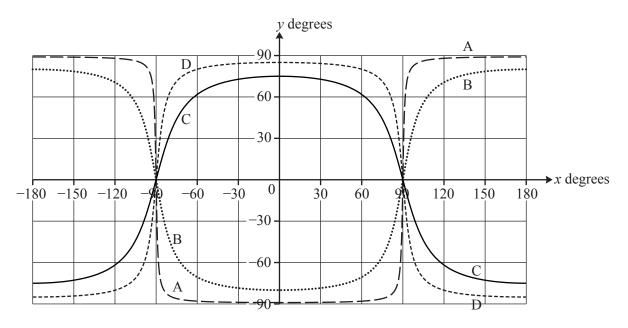


Fig. 6

3	10° north:
	1° north:
	5° south:
	15° south:

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4 In lines 94 and 95 the article sa
-------------------------------------

"Fig. 8 shows you that at latitude  $60^\circ$  north the terminator passes approximately through time +9 hours and -9 hours so that there are about 18 hours of daylight."

[4]

Use Equation (4) to check the accuracy of the figure of 18 hours.

4	
,	
,	

5 (i)	Use Equation (3) to calculate the declination of the sun on February 2nd.	[3]
(ii)	The town of Boston, in Lincolnshire, has latitude $53^{\circ}$ north and longitude $0^{\circ}$ .	
	Calculate the time of sunset in Boston on February 2nd.	
	Give your answer in hours and minutes using the 24-hour clock.	[4]
5 (i)		
5 (ii)		

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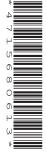


# Thursday 13 June 2013 – Morning

# **A2 GCE MATHEMATICS (MEI)**

**4754/01** Applications of Advanced Mathematics (C4)

**INSTRUCTIONS** 



The examination is in two parts:

Paper A (1 hour 30 minutes)
Paper B (up to 1 hour)

Supervisors are requested to ensure that Paper B is not issued until Paper A has been collected in from the candidates.

Centres may, if they wish, grant a supervised break between the two parts of this examination.

Paper B should not be attached to the corresponding paper A script. For Paper A only the candidates' printed answer books, in the same order as the attendance register, should be sent for marking; the question paper should be retained in the centre or recycled. For Paper B only the question papers, on which the candidates have written their answers, should be sent for marking; the insert should be retained in the centre or recycled. Any additional sheets used must be carefully attached to the correct paper.

For Paper B (Comprehension) only.

A standard English dictionary is allowed for the comprehension.

(Dictionaries to be used in the examination must be thoroughly checked before the examination.) Full regulations are in the JCQ Regulations and Guidance booklet.

This notice must be on the Invigilator's desk at all times during the morning of Thursday 13 June 2013.



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# **Thursday 13 June 2013 - Morning**

# **A2 GCE MATHEMATICS (MEI)**

**4754/01B** Applications of Advanced Mathematics (C4) Paper B: Comprehension **INSERT** 



# **INFORMATION FOR CANDIDATES**

- This Insert contains the text for use with the questions.
- This document consists of 8 pages. Any blank pages are indicated.

### INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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# **Day-Night Maps**

On many inter-continental flights you will see a *day-night map* displayed, like that in Fig. 1. It shows those parts of the earth that are in daylight and those that are in darkness. Such maps usually show the position of the aeroplane. They also, as in this case, often show the point that is immediately under the sun; at that point the sun is directly overhead.



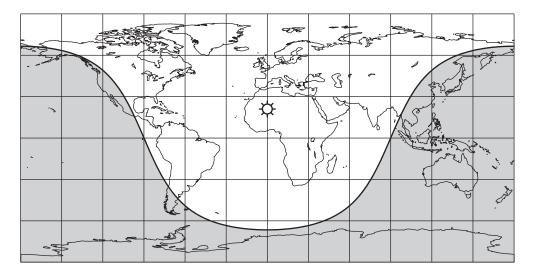


Fig. 1

Fig. 1 shows the day-night map when it is mid-day in the United Kingdom on mid-summer's day in the northern hemisphere.

### **Modelling assumptions**

Fig. 2 illustrates the earth as a 3-dimensional object being illuminated by the sun. At any time the sun is shining on approximately half of the earth's surface but not on the other half. The two regions are separated by a circle on the earth's surface called the *terminator*. This is represented in Fig. 1 as the curve separating the light and dark regions.



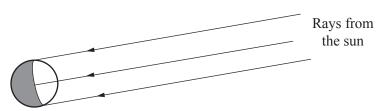


Fig. 2

In this article, a number of modelling assumptions are made to simplify the situation.

- The sun is taken to be a point so that at any time it is either above the horizon or below it, but never partly above and partly below.
- All light rays coming from the sun to the earth are parallel.
- The effects of refraction (bending of the light by the earth's atmosphere) are negligible.
- The earth is a perfect sphere.

The effects of these assumptions are that, at any time, exactly (and not approximately) half of the earth's surface is being illuminated by the sun and there is no twilight; at any place, it is either day or night. None of the assumptions is actually quite true, but they are all close enough to provide a good working model.

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Fig. 3, below, is in two dimensions; it shows a section of the earth through its centre, O, in the same plane as the sun.

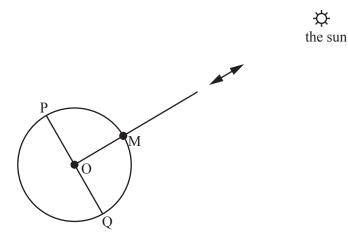


Fig. 3

The line from the centre of the earth to the sun cuts the surface of the earth at the *sub-solar point*, M. If you were standing at M you would see the sun directly overhead. On a typical day-night map there is a picture of the sun at the sub-solar point.

The points P and Q are on the terminator:  $\angle POM = \angle QOM = 90^{\circ}$ . If you were standing at P or Q it would either be the moment of sunrise or the moment of sunset for you.

# The cartesian equation of the terminator

Fig. 4 shows the map of the world with x- and y-axes superimposed.

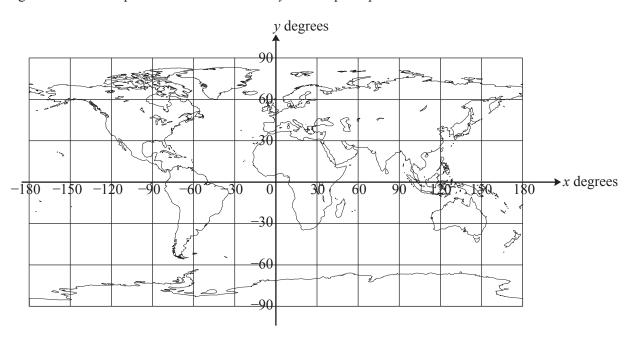


Fig. 4

On these axes:

- x represents longitude, going from  $-180^{\circ}$  (180° west) to  $+180^{\circ}$  (180° east),
- y represents latitude, going from  $-90^{\circ}$  (90° south) to  $+90^{\circ}$  (90° north).

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25

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The lines parallel to the x-axis are called lines of latitude. On the earth's surface they are actually circles. The x-axis itself is the equator.

35

The lines parallel to the y-axis are called lines of longitude, or *meridians*. Each meridian is actually a semicircle along the earth's surface joining the north pole and the south pole. The y-axis is the zero meridian; it passes through Greenwich in London and so is called the Greenwich meridian.

The y-coordinate of the sub-solar point is its latitude, measured in degrees, and is called the *declination* of the sun. In this article the declination of the sun is denoted by  $\alpha$ . During a year, the value of  $\alpha$  varies between  $+23.44^{\circ}$  on mid-summer's day in the northern hemisphere and  $-23.44^{\circ}$  on mid-winter's day. Fig. 1 shows the situation on mid-summer's day, when the sun is at its most northerly, and so  $\alpha = 23.44^{\circ}$ .

40

Using these axes, it is possible to show that the equation of the terminator on the map, for the time and day shown in Fig. 1, can be written as the cartesian equation

$$\tan y = -2.306 \cos x$$
. (1)

This can also be written as

$$y = \arctan(-2.306\cos x)$$
.

Fig. 5 shows this curve.

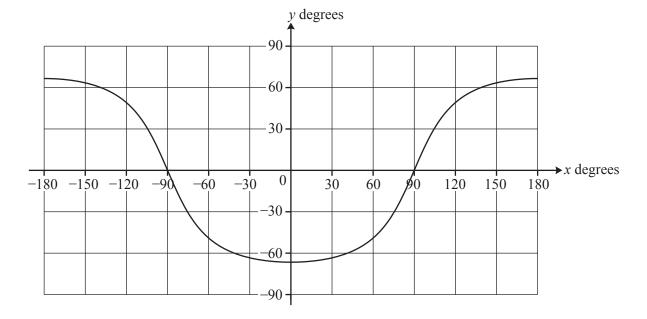


Fig. 5

When this curve is superimposed on the map of the world, and the correct region is shaded, the day-night map in Fig. 1 is produced.

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The terminator is a circle on the earth's surface and so it is quite surprising that the curve in Fig. 5 looks nothing like a circle. There are two points to be made.

- The map in Fig. 4 is formed from a cylinder that has been cut along the line  $x = \pm 180$  and laid flat. x = +180 and x = -180 are the same line. So the curve is continuous.
- The earth is a sphere and representing it on a cylinder causes distortion; this affects the shape of the curve on the map. In particular, the polar regions become very distorted, and, along with them, the circular shape of the terminator.

The representation used to draw a sphere on a flat sheet of paper is called the map's *projection*. There are very many different map projections; the one used for day-night maps is called *equirectangular cylindrical* (or *plate carrée*).

60

### The terminator at other times and on other dates

So far, only one time and day of the year has been considered, mid-day on mid-summer's day in the northern hemisphere when the declination of the sun is  $+23.44^{\circ}$ . What about other times of day? And other days of the year?

The answer to the question about different times of day is that, as the earth rotates, the sub-solar point moves along its circle of latitude and the terminator moves with it, keeping the same shape.

65

The question about other days of the year relates to the declination of the sun. On mid-summer's day,  $\alpha = 23.44^{\circ}$ . Equation (1) is  $\tan y = -2.306 \cos x$ ; the number 2.306 arises because

$$\frac{1}{\tan 23.44^{\circ}} = 2.306.$$

For a general value of  $\alpha$ , the number 2.306 is replaced by

70

and so the equation of the terminator can be written in general form as

$$\tan y = -\frac{1}{\tan \alpha} \cos x. \tag{2}$$

Equation (2) makes it possible to draw a graph illustrating the terminator for any possible declination of the sun. Fig. 6 shows the terminator in the cases where the sun has declination  $10^{\circ}$  north,  $1^{\circ}$  north,  $5^{\circ}$  south and  $15^{\circ}$  south. In each case the time is mid-day on the Greenwich meridian.

75

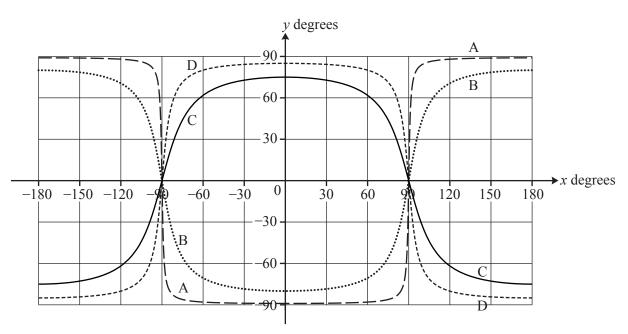
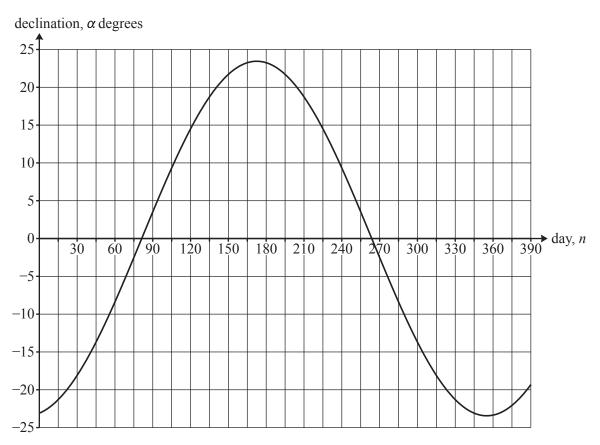


Fig. 6

### The declination of the sun

In Fig. 1 the sun is at its most northerly point with declination  $+23.44^{\circ}$ . On mid-winter's day its declination is  $-23.44^{\circ}$ . To good approximation, the value of the declination follows a sine curve between these two values, as shown in Fig. 7. Day 1 is January 1st.

80



**Fig. 7** 

The equation of the curve in Fig. 7 is

$$\alpha = -23.44 \times \cos\left(\frac{360}{365} \times (n+10)\right). \tag{3}$$

(This approximation is based on the modelling assumption that the orbit of the earth around the sun is a circle; it is actually an ellipse.)

Hours of daylight 85

If you travel north when it is summer in the northern hemisphere, you will notice that the days become longer and the nights shorter. The graph of the terminator allows you to see how this happens.

The earth rotates on its axis once every day. So it turns through  $360^{\circ}$  every 24 hours or  $15^{\circ}$  per hour. So every  $15^{\circ}$  of longitude (ie along the *x*-axis in Fig. 5) corresponds to 1 hour of time.

Fig. 8 is the same as Fig. 5 (the declination of the sun is  $+23.44^{\circ}$ ) but the horizontal axis represents the time difference from Greenwich, measured in hours, rather than longitude measured in degrees.



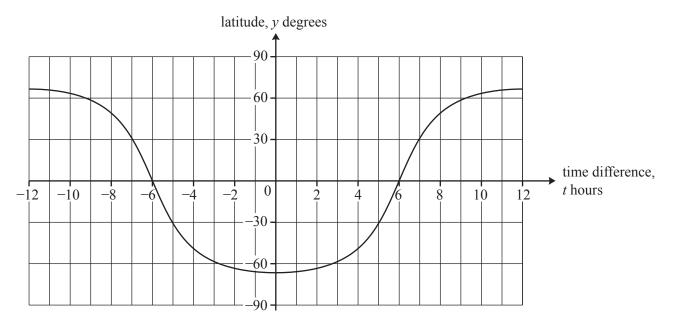


Fig. 8

The graph in Fig. 8 shows y against t. Since x = 15t, the equation of the terminator can be written as

$$\tan y = -\frac{1}{\tan \alpha} \cos(15t). \tag{4}$$

Fig. 8 shows you that at latitude 60° north the terminator passes approximately through time +9 hours and –9 hours so that there are about 18 hours of daylight. Oslo has latitude 60° north.

95

You can also see that at latitude 30° north there are about 14 hours of daylight on this day of the year. Cairo has latitude 30° north.

So Oslo has about 4 more hours of daylight than Cairo on this day.

At the start of the article, it was stated that one effect of the modelling assumptions is to ignore twilight. This is the time when the sun is just below the horizon. The effect of twilight is particularly noticeable in places with high latitudes, for example Oslo, in the summer so that it is nearly light for even longer.

100



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**GCE** 

# **Mathematics (MEI)**

Advanced GCE

Unit 4754B: Applications of Advanced Mathematics: Paper B

# **Mark Scheme for June 2013**

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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# **Annotations**

Annotation in scoris	Meaning
✓and <b>x</b>	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
۸	Omission sign
MR	Misread
Highlighting	
Other abbreviations in	Meaning
mark scheme	
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

## Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

#### $\mathbf{M}$

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

#### A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

#### В

Mark for a correct result or statement independent of Method marks.

 $\mathbf{E}$ 

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep \*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.
- g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last

- (complete) attempt and ignore the others.
- NB Follow these maths-specific instructions rather than those in the assessor handbook.
- h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.
  - Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question		Answer	Marks	Guidance				
1		Point R marked correctly at (-60, 45)		half way up the relevant square	Need labelling with letters or			
		Point M marked correctly at (0, 20)	B1	Generously applied if above half way	co-ordinates oe (condone one not labelled if other is labelled (and no others marked))			
				If unclear, B0B0.	SC B1 BOD both marked (and no others) and no labelling.			
			[2]					
2		For 30° S, 10 hours daylight	B1	cao soi	10 only			
		For 60° S, 5.5 hours daylight.	B1	allow 5< <i>t</i> <6 soi	not 5 or 6			
		Difference 4.5 hours	B1ft	10- t dependent on both previous B marks soi				
					SC(1) allow B3 for 4 <difference in="" length<5<br="">(without wrong working)</difference>			
					<b>SC(2)</b> allow <b>B2</b> if uses t=5 or t=6 eg, 10,6,4 www			
					SC(3) If calculates (using the equation (4)) can obtain all three marks.			
					Approximate values are 10.07, 5.51, 4.56.			
			[3]		(May also work with 5 and 2.5 <t<3 5="" at="" b1="" dependent="" doubled="" doubling<="" eg="" end="" if="" later="" on="" th=""></t<3>			
					2.6 B1dependent on later doubling (5-2.6) x2=4.8 B1			
		IT IS ESSENTIAL TO CHECK & ANNOTAT	ΓE PAGE <mark>S ATTA</mark>	CHED TO QUESTIONS 101, 102, 1	.03			

Question		Answer	Marks	Guid	lance
3		10° north is B			
		1° north is A	B2	All four answers correct	
		5° south is D			
		15° south is C		SC B1 Any two answers correct	
			[2]		
4		$\tan y = -\frac{1}{\tan \alpha} \cos(15t)$			
		$\alpha = 23.44^{\circ}, y = 60^{\circ}, t \text{ is to be found}$			
		$\cos(15t) = -0.7509$	M1	substitute in formula and attempt to	
		15 <i>t</i> = 138.6737		solve (as far as 15 <i>t</i> =invcos)oe	
		t = 9.2449	A1	accept 9.2 or better	
		Daylight hours are $2 \times 9.2449 = 18.4898$	DM1	doubling, dependent on first M1	
		So 18.5 hours (to 3s.f.)	A1	or approx 18 hours www (accept 18.4898,18.489,18.49,18.5 or 18.4 (from 2x9.2),18.48)	any reasonable accuracy or stating error is approx 0.49 oe
		OR			any reasonable accuracy
		Using $t=9$ M1 $t=18/2=9$ and substituted		t=18/2=9 and substituted in formula	any reasonable accuracy
		$\alpha$ =23.44, $t$ =9, $y$ is to be found			
		$\tan y = 1.6309$	DM1	and attempt to solve (as far as	
				y= inv tan constant) oe	
		<i>y</i> =58.485° so approx 60°	A2	or 58.49°/58.5°/approx 60° www	
			[4]		

Question		on	Answer	Marks	Guidance				
5	(i)		$\alpha = -23.44 \times \cos\left(\frac{360}{365} \times (n+10)\right)$						
			On February 2nd, $n = 31 + 2 = 33$	B1	calculate $n=31+2=33$ (days in Jan + Feb) soi	SC B1 condone 30+2			
			$\alpha = -23.44 \times \cos\left(\frac{360}{365} \times 43\right)$	M1	substitution of their $n+10$ in equation (3) and attempt to evaluate	Where <i>n</i> =31,32,33,34 only			
			a = -17.31	A1	or -17.306 or rounds to -17.3				
						NB <i>n</i> = 32+10 gives -17.576 gaining B1M1A0			
				[3]					
5	(ii)		$\tan y = -\frac{1}{\tan \alpha} \cos(15t)$						
			$\tan 53 = -\frac{1}{\tan(-17.306)} \times \cos(15t)$	M1	use of <b>their</b> $\alpha$ in equation (4)				
			$t = \frac{1}{15}\arccos(-\tan 53 \times \tan(-17.306))$	DM1	making t the subject				
			t = 4.3717	A1	4.37 or better	SC ft from -17.576 Obtains A1ft for 4.343 (or 4.34)			
			Sunset is at 12 hrs + 4 hours 22 minutes, and so 16:22 hrs	A1	cao	And then A1ft for 16:21 (or 16:20)			
				[4]					

**OCR (Oxford Cambridge and RSA Examinations)** 1 Hills Road Cambridge **CB1 2EU** 

# **OCR Customer Contact Centre**

# **Education and Learning**

Telephone: 01223 553998 Facsimile: 01223 552627

Email: general.qualifications@ocr.org.uk

# www.ocr.org.uk

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Head office

Telephone: 01223 552552 Facsimile: 01223 552553







**GCE** 

# **Mathematics (MEI)**

Advanced GCE A2 7895-8

Advanced Subsidiary GCE AS 3895-8

# **OCR Report to Centres**

**June 2013** 

# 4754 Applications of Advanced Mathematics (C4)

#### **General Comments**

This paper was of a similar standard to previous years.

The questions were accessible to candidates of all abilities who were able to demonstrate their skills. There were few very low scores and also few very high scores, with full marks obtained by a few candidates. The higher scoring candidates were able to show their skills - particularly in Paper A questions 3, 6(iii) and 7.

The comprehension, Paper B, was well understood and most candidates scored good marks here.

As in previous years, many candidates lost unnecessary marks through poor algebra. Some particularly common such examples being:

• 
$$\frac{1}{3(1+x)} = 3(1+x)^{-1} = 3(1-x+x^2...)$$

• 
$$\left(e^{x/5} + e^{-x/5}\right)^2 = e^{2x/5} + e^{-2x/5}$$

• 
$$\csc x + 5 \cot x = 3 \sin x \Rightarrow \csc^2 x + 25 \cot^2 x = 9 \sin^2 x$$

These, and other algebraic errors, are detailed later in this Report.

Sign errors also continue to be a common cause of an unnecessary loss of marks.

In contrast, however, it was very pleasing to note that, unlike in previous years, few candidates failed to put a constant of integration. Examiners would now like to encourage candidates to change the constant when say multiplying through by 2 rather than renaming their constant as c at every stage.

Candidates should be reminded that when they are asked to 'Show' they need to show all stages of working. This is improving, but it is disappointing when marks are lost in this way.

Centres should be reminded that Papers A and B are marked separately and so supplementary sheets should be attached to the appropriate paper.

#### **Comments on Individual Questions**

#### Paper A

- 1)(i) Whilst almost all candidates knew the general method for expressing the given fraction
- 1)(ii) in partial fractions, there were a surprising number of numerical errors.

Most candidates were able to use the binomial expansion correctly although there were sign errors - often from using (-2x) as (2x).

The most common error-which was very common- was using

$$\frac{1}{3(1+x)} = 3(1+x)^{-1} = 3(1-x+x^2...) = 3-3x+3x^2 \quad \text{and similarly for } \frac{1}{3(1-2x)}.$$

The other frequent error was in the validity. Some candidates omitted this completely but many others failed to combine the validities from the two expansions, or failed to choose the more restrictive option.

2) Many candidates scored full marks when showing that the trigonometric equation could be rearranged as a quadratic and then solving it.

Where there were errors, these were usually in the first part when trying to establish the given result. Errors included failing to use the correct trigonometric identities, failing to use  $\sin^2\theta + \cos^2\theta = 1$  or squaring the original expression term by term. Few candidates would say x+3=7 so  $x^2+9=49$  and yet they happily square cosec x+5cot x=3sin x term by term.

Those who were unable to complete the first part sensibly then proceeded to solve the quadratic equation. Few errors were seen here. Occasionally the final solution was incorrect and few candidates offered additional incorrect solutions.

3) There were some good explanations with appropriate triangles in the first part.

However, too many candidates felt it was enough to only give the information given in the question and this was not sufficient. More was needed than, for example, a right-angled triangle with lengths of 1, 1 and  $45^{\circ}$  to show that tan  $45^{\circ}=1$ . It was necessary to clearly show the triangle was isosceles by giving the other angle or showing that the hypotenuse was  $\sqrt{2}$ , or equivalent. Some made errors when calculating the other lengths in both triangles. Some good candidates failed to score here seemingly being unfamiliar with where these identities came from.

The second part started well for most candidates, who usually used the correct compound angle formula, (although there were a few who thought that tan75°=tan45°+tan30°) and made the first substitution. Thereafter, this question gave the opportunity for candidates to show that they could eliminate fractions within fractions and rationalise the denominator. This was a good discriminator for the higher scoring candidates. A few candidates abandoned their attempt at half way and equated

$$\frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}$$
 at that stage to the given answer 2+ $\sqrt{3}$ .

4) Most candidates scored high marks throughout this question.

In (i) the most common error was to omit r =at the start. Few candidates would write, for instance, y = x+3 without the y but the r =is too often omitted from vector equations.

In (ii) errors were usually numerical and in (iii) they were either numerical errors or the wrong vectors.

5) Most candidates scored the first four marks by forming the equations and solving them.

Marks were usually lost both when candidates failed to show their solutions worked in all three equations or failed to realise that O, A, B and C must all lie on the same plane for the final mark.

6)(i) Most candidates scored all four marks when solving the differential equation. It was pleasing to see so few candidates failing to include the constant of integration. Some candidates, however, tried to work backwards from the answer, or wrote  $v^2 = -4x^2 + c$  without showing from where it came. The answer was given in this case so stages of working were needed.

Whilst, on this occasion, examiners condoned the change of constant candidates should be encouraged to change their constant when appropriate in the future and not use *c* twice to mean different things within the same question.

- 6)(ii) Most candidates obtained the mark for verifying that *x* =1. Many others also scored the following three marks but some had the incorrect coefficients when differentiating and only had the correct coefficient in the second term when working backwards from the answer, 4.
- 6)(iii) This part was rarely answered completely successfully. Most candidates understood that the 'R' method was needed and scored the first three marks. This was the modal mark.

Calculus was needed in this question. The candidates were asked to find the constant  $\alpha$ , and t is a time to be combined in  $(2t-\alpha)$  so answers given in radians were required. The use of degrees here was a very common mistake. Many candidates then differentiated their angles in degrees and obtained no marks for v.

The last two marks were obtained only rarely but they were a good differentiator for the able candidates. Use of unspecified  $\alpha$ , or in degrees or radians was allowed in this last part. Some candidates had difficulty as they had found both x and v separately using the 'R' method and so had different values for their angles. Others realised the problem and were able to use trigonometric identities to change their v (or x) to have the same angle.

- 6)(iv) The majority of candidates scored the first two marks.. It was disappointing that candidates did not realise their mistake in part (ii) when they obtained an answer of time, t=31.7 degrees.
- 7)(i) Some candidates were able to score full marks here with ease. Some candidates gave their answers as coordinates instead of lengths and others found OC instead of AC. There were also, however, some very confused and unclear methods used and many candidates lost marks having failed to use u = 1 and u = 10 or equivalent.
- 7)(ii) Most candidates understood that they needed to find and divide dy/du by dx/du. There was some very poor algebra when attempting to simplify  $\frac{1-1/u^{-2}}{5/u}$ . The derivatives were also frequently wrong-often including ln u. Many candidates stopped at this stage or substituted u =10 in their derivative and then stopped. Some candidates, who were able to score marks in the following stages, failed to realise that they could invert the derivative and quickly find the answer. Some used the gradient, 1.98, to form an equation of a straight line and find its intercept with the x axis. Other candidates, unfortunately, felt they could use 1.98 as a hypotenuse in the triangle with AC-with no success.
- 7)(iii) This was well understood but candidates lost marks by giving insufficient working when establishing a given result.

# OCR Report to Centres - June 2013

7)(iv) There were some excellent solutions here but the majority only scored one mark. This was awarded to those who correctly showed us their intention to find  $V = \int \pi (e^{x/5} + e^{-x/5})^2 dx$ . However, the majority could not expand this bracket. Usually it was thought to equal  $e^{2x/5} + e^{-2x/5}$  but other incorrect options were seen, including powers such as  $x^2/25$ . For those who did expand the bracket correctly, other errors followed-either using the wrong upper limit, failing to substitute the lower limit or, more commonly integrating either 2 as 0, or more particularly  $e^{2x/5}as\frac{2e^{2x/5}}{5}$  and similarly for the other term.

## Paper B

- 1) Answers were often correct but surprisingly many, and various, incorrect positions were seen. A number of candidates only used the letter *R* or *M* to indicate their points where the addition of a cross or dot would have made their position clearer.
- 2) Most candidates had the right idea but some were inaccurate with 10, 6, 4 being the most common alternative solution.
- 3) The graphs were usually identified correctly although there were also many guesses. The response tended to be either fully correct or all wrong.
- 4) Most candidates substituted  $\alpha = 60^{\circ}$  and found t = 9.2449 or similar and then the majority multiplied it by 2 to compare it with 18. A few worked backwards from t = 9 to reach approximately  $60^{\circ}$  when substituting in the appropriate equation, and were given full credit.
- 5(i) Most candidates found  $\alpha = -17.31$  as required. A few chose to use the number of days in January as 30 and lost one mark. Those who thought February was the first or third month received no credit.
- 5)(ii) Some candidates carelessly lost the negative sign in their angle or the negative sign in the formula and so lost unnecessary marks.

Many correctly obtained t=4.37 but not all converted this correctly to the 24 hour clock. 16:37 was commonly seen.

Candidates were able to follow through for full marks from the Special Case in part(ii)



# Unit level raw mark and UMS grade boundaries June 2013 series AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

GCE Mathematics (MEI)		Max Mark	а	b	С	d	е	u
4751/01 (C1) MEI Introduction to Advanced Mathematics	Raw	72	62	56	51	46	41	0
	UMS	100	80	70	60	50	40	0
4752/01 (C2) MEI Concepts for Advanced Mathematics	Raw	72	54	48	43	38	33	0
47F0/04 (OO) MELMathada (and harran I Mathada (and harran I Mathad	UMS	100	80	70	60	50	40	0
4753/01 (C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper 4753/02 (C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw Raw	72 18	58 15	52 13	46 11	40	33 8	0 0
4753/82 (C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9 9	8	0
4753 (C3) MEI Methods for Advanced Mathematics with Coursework	UMS	100	80	70	60	50	40	0
4754/01 (C4) MEI Applications of Advanced Mathematics	Raw	90	66	59	53	47	41	0
	UMS	100	80	70	60	50	40	0
4755/01 (FP1) MEI Further Concepts for Advanced Mathematics	Raw	72	63	57 70	51	45 50	40	0
4756/01 (FP2) MEI Further Methods for Advanced Mathematics	UMS	100	80 61	70	60	50	40	0
4/56/01 (FP2) MEI Further Methods for Advanced Mathematics	Raw UMS	72 100	61 80	54 70	48 60	42 50	36 40	0 0
4757/01 (FP3) MEI Further Applications of Advanced Mathematics	Raw	72	60	52	44	36	28	0
4707701 (11 0) WETT dittle! Applications of Advanced Mathematics	UMS	100	80	70	60	50	40	0
4758/01 (DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	62	56	51	46	40	0
4758/02 (DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758/82 (DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4758 (DE) MEI Differential Equations with Coursework	UMS	100	80	70	60	50	40	0
4761/01 (M1) MEI Mechanics 1	Raw	72	57	49	41	33	25	0
4762/01 (M2) MEI Mechanics 2	UMS	100 72	80 50	70 43	60 36	50 29	40 22	0
4762/01 (MZ) MET Mechanics 2	Raw UMS	100	80	43 70	60	29 50	40	0 0
4763/01 (M3) MEI Mechanics 3	Raw	72	64	56	48	41	34	0
	UMS	100	80	70	60	50	40	0
4764/01 (M4) MEI Mechanics 4	Raw	72	56	49	42	35	29	0
	UMS	100	80	70	60	50	40	0
4766/01 (S1) MEI Statistics 1	Raw	72	55	48	41	35	29	0
	UMS	100	80	70	60	50	40	0
4767/01 (S2) MEI Statistics 2	Raw UMS	72 100	58 80	52 70	46 60	41 50	36 40	0 0
4768/01 (S3) MEI Statistics 3	Raw	72	61	55	49	50 44	39	0
4700/01 (00) MEI Otalistics 3	UMS	100	80	70	60	50	40	0
4769/01 (S4) MEI Statistics 4	Raw	72	56	49	42	35	28	0
	UMS	100	80	70	60	50	40	0
4771/01 (D1) MEI Decision Mathematics 1	Raw	72	58	52	46	40	35	0
	UMS	100	80	70	60	50	40	0
4772/01 (D2) MEI Decision Mathematics 2	Raw	72	58	52	46	41	36	0
ATTO/OA (DO) MEI Decision Mathematica Commutation	UMS	100	80	70	60	50	40	0
4773/01 (DC) MEI Decision Mathematics Computation	Raw UMS	72 100	46 80	40 70	34 60	29 50	24 40	0 0
4776/01 (NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	56	50	44	38	31	0
4776/02 (NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776/82 (NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
4776 (NM) MEI Numerical Methods with Coursework	UMS	100	80	70	60	50	40	0
4777/01 (NC) MEI Numerical Computation	Raw	72	55	47	39	32	25	0
	UMS	100	80	70	60	50	40	0
4798/01 (FPT) Further Pure Mathematics with Technology	Raw UMS	72 100	57 80	49 70	41 60	33 50	26 40	0 0
GCE Statistics (MEI)	OIVIO	100	00	70	00	30	40	
		Max Mark	а	b	С	d	е	u
G241/01 (Z1) Statistics 1	Raw	72	55	48	41	35	29	0
	UMS	100	80	70	60	50	40	0
G242/01 (Z2) Statistics 2	Raw	72	55	48	41	34	27	0
C242/04 /72) Statistics 2	UMS	100	80	70	60	50	40	0
G243/01 (Z3) Statistics 3	Raw UMS	72 100	56 80	48 70	41 60	34 50	27 40	0 0
	OIVIO	100	00	70	00	50	40	U